

Remote two-qubit state creation and its robustness

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Abstract

We consider the problem of remote two-qubit state creation using the two-qubit excitation pure initial state of the sender. The communication line is based on the optimized boundary controlled chain with two pairs of properly adjusted coupling constants. The creation of the two-qubit Werner state is considered as an example. We also study the effects of imperfections of the chain on the state-creation.

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I. INTRODUCTION

Information processing devices, be they classical or quantum in nature, consist of storage elements, gates, and interconnections along which information can be transferred. In a quantum device information is contained in the quantum state of one or more quantum bits, or qubits. For the transfer of information between different locations several concepts have been developed. Quantum state transfer [1–8] denotes the transport of an existing state to a different place by the dynamics of an underlying Hamiltonian with or without time-dependent control parameters. In contrast, remote state creation [9–15] means the preparation of a desired state in a desired location by manipulations performed at an earlier time in a different location. Although this technique was first realized in quantum optics [12, 13, 15], systems of coupled spins have also been studied in this context and may be used for short-distance communication, for instance, within a quantum information processing device.

The principal setup for remote state creation used in our previous papers [16–20] is as follows. There are two small sets of qubits called sender and receiver, respectively, connected by a chain of other qubits serving as a transmission line. The state of the sender is characterized by a set of parameters which may be adjusted in order to create the desired state in the receiver subsystem. These are the control parameters. The state of the receiver is also characterized by a set of parameters, the target parameters. The possible states of the receiver occupy a certain region in the space of target parameters. In recent papers we have studied various aspects of the remote creation of single-qubit states: the creatable region [16], i.e. the region in the target parameter space corresponding to states which can actually be remotely created by adjusting the control parameters, the creation of states from pure one-excitation initial states [17], the enhancement of performance by unitary transformations on the receiver side [18], the remote creation of quantum correlations (discord) [19] as well as the control of polarization and coherence intensity of the created state [20].

For transmission lines (spin chains) designed for perfect state transfer [2, 4] the creatable region covers all possible states of the receiver. However, these chains require careful adjustment of all couplings along the chain which is difficult and sensitive towards “manufacturing errors”, small imperfections in the coupling constants which reduce the perfect state transfer to a high-probability state transfer. Since a high-probability state transfer may also be

achieved much more simply by boundary-controlled [5, 6] or optimized boundary controlled chains [21–24], we study this type of systems in the present paper.

Our goal here is the remote creation of two-qubit states. For that purpose we have to employ initial states with two excitations instead of the one-excitation states studied previously. While the creatable region for a one-qubit receiver is essentially (i.e. up to an unimportant phase) defined by the values of two parameters and thus can be shown in a single picture [16, 17], this becomes much more complicated for a two-qubit receiver. There, the density matrix depends on 15 parameters and thus is difficult to visualize. Therefore we have to employ other means of controlling the creatable states. We show that the communication line is characterized by a finite number of Hamiltonian-dependent parameters and this number is defined by the dimensions of the sender and receiver and does not depend on the whole length of the chain.

The structure of this paper is as follows. In Sec.II we present the general protocol of two-qubit state creation without specifying the interaction Hamiltonian. Then, in Sec.III, we apply this protocol to a particular spin-1/2 model governed by the XY Hamiltonian and describe the creation of the Werner state as an example. The effect of imperfections of the Hamiltonian on the state creation is considered in Sec. IV. Sec.V contains concluding remarks .

II. GENERAL PROTOCOL OF REMOTE TWO-QUBIT STATE CREATION

Our protocol employs a N -node spin-1/2 chain and assumes that the evolution of this chain is governed by a Hamiltonian commuting with the z -projection of the total spin momentum I_z : $[H, I_z] = 0$. Then the spin dynamics can be described in the subspace spanned by the vectors

$$|0\rangle, |k\rangle, k = 1, \dots, N, |nm\rangle, n = 1, \dots, N-1, m = 2, \dots, N, m > n, \quad (1)$$

whose dimensionality is $N + 1 + \binom{N}{2} = \frac{1}{2}(N^2 + N + 2)$. In eqs.(1), $|k\rangle$ means the state with the k th spin excited, $|nm\rangle$ means the state with the n th and m th spins excited, and $|0\rangle$ is the state without excitations.

A. General form of the two-excitation initial state

In this section we do not restrict the size N_S of the sender and consider the general form of the two-qubit initial state:

$$|\Psi_0\rangle = a_0|0\rangle + \sum_{i=1}^{N_S} a_i|i\rangle + \sum_{\substack{i,j=1 \\ j>i}}^{N_S} a_{ij}|ij\rangle, \quad (2)$$

with normalization

$$|a_0|^2 + \sum_{i=1}^{N_S} |a_i|^2 + \sum_{\substack{i,j=1 \\ j>i}}^{N_S} |a_{ij}|^2 = 1. \quad (3)$$

Here a_i and a_{jk} are the amplitudes of the initial state. The initial state $|\Psi_0\rangle$ evolves according to the Schrödinger equation as $|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle$, where H is the Hamiltonian governing the spin-dynamics.

B. State of the two-qubit receiver and its characteristics

The density matrix ρ^R of the state of the two-qubit receiver is reducible from the density matrix $|\Psi(t)\rangle\langle\Psi(t)|$ of the state of the whole system by tracing over all nodes except for the two nodes of the receiver. In the basis $|0\rangle, |N-1\rangle, |N\rangle, |(N-1)N\rangle$, the matrix ρ^R reads:

$$\rho^R(t) = \text{Tr}_{1,\dots,N-2}|\Psi(t)\rangle\langle\Psi(t)| = \quad (4)$$

$$\begin{pmatrix} 1 - \rho_{N-1;N-1} - \rho_{N;N} - \rho_{(N-1)N;(N-1)N} & \rho_{0;N-1} & \rho_{0;N} & \rho_{0;(N-1)N} \\ \rho_{0;N-1}^* & \rho_{N-1;N-1} & \rho_{N-1;N} & \rho_{N-1;(N-1)N} \\ \rho_{0;N}^* & \rho_{N-1;N}^* & \rho_{N;N} & \rho_{N;(N-1)N} \\ \rho_{0;(N-1)N}^* & \rho_{N-1;(N-1)N}^* & \rho_{N;(N-1)N}^* & \rho_{(N-1)N;(N-1)N} \end{pmatrix}.$$

In (4), all elements $\rho_{ij} \equiv \rho_{ij}(t)$ are expressed in terms of the transition amplitudes f_i and f_{ij} from the initial state to the basis states:

$$f_i(t) = \langle i|\Psi(t)\rangle = \sum_{k=1}^{N_S} a_k p_{i;k}(t), \quad (5)$$

$$f_{ij}(t) = \langle ij|\Psi(t)\rangle = \sum_{\substack{n,m=1 \\ m>n}}^{N_S} a_{nm} p_{ij;nm}(t), \quad (6)$$

where

$$p_{i;k} = \langle i | e^{-iHt} | k \rangle, \quad p_{ij;nm} = \langle ij | e^{-iHt} | nm \rangle \quad (7)$$

are the transition amplitudes between the basis vectors. In addition, we assume that the state $|0\rangle$ corresponds to zero energy, hence $f_0 = \langle 0 | \Psi(t) \rangle = \langle 0 | \Psi_0 \rangle = a_0$, and a_0 is a real number. Notations (5) and (6) show that (i) the transition amplitudes f_i and f_{ij} are linear functions of the initial state amplitudes a_i and a_{ij} , and (ii) the dependence on the Hamiltonian (in particular, on the coupling constants characterizing the transmission line) is confined to the parameters $p_{i;k}$ and $p_{ij;nm}$. Hence, these parameters can be considered fixed for a given transmission line while the amplitudes a_i and a_{ij} serve as control parameters of the sender which can be varied in order to create the desired state of the receiver. We collect these parameters in the list a :

$$a = \{a_0, a_k, a_{nm} : k = 1, \dots, N_S, n = 1, \dots, N_S - 1, m = 2, \dots, N_S, m > n\}. \quad (8)$$

This list consists of $N_S^2 + N_S$ independent real parameters.

The direct calculation of the elements of the density matrix ρ^R yields:

$$\rho_{0;N-1} = f_0 f_{N-1}^* + \sum_{i=1}^{N-2} f_i f_{i(N-1)}^* = a_0 \sum_{k=1}^{N_S} p_{N-1;k}^* a_k^* + \sum_{\substack{k,n,m=1 \\ m>n}}^{N_S} P_{N-1;knm} a_k a_{nm}^*, \quad (9)$$

$$\rho_{0;N} = f_0 f_N^* + \sum_{i=1}^{N-2} f_i f_{iN}^* = a_0 \sum_{k=1}^{N_S} p_{N;k}^* a_k^* + \sum_{\substack{k,n,m=1 \\ m>n}}^{N_S} P_{N;knm} a_k a_{nm}^*, \quad (10)$$

$$\rho_{0;(N-1)N} = f_0 f_{(N-1)N}^* = a_0 \sum_{\substack{n,m=1 \\ m>n}}^{N_S} p_{(N-1)N;nm}^* a_{nm}^*, \quad (11)$$

$$\rho_{N-1;N-1} = f_{N-1}f_{N-1}^* + \sum_{i=1}^{N-2} f_{i(N-1)}f_{i(N-1)}^* = \quad (12)$$

$$\sum_{n,m=1}^{N_S} p_{N-1;n}p_{N-1;m}^*a_n a_m^* + \sum_{\substack{k,l,n,m=1 \\ m>n,l>k}}^{N_S} P_{(N-1)(N-1);klm}a_{kl}a_{nm}^*,$$

$$\rho_{N-1;N} = f_{N-1}f_N^* + \sum_{i=1}^{N-2} f_{i(N-1)}f_{iN}^* = \quad (13)$$

$$\sum_{n,m=1}^{N_S} p_{N-1;n}p_{N;m}^*a_n a_m^* + \sum_{\substack{k,l,n,m=1 \\ m>n,l>k}}^{N_S} P_{(N-1)N;klm}a_{kl}a_{nm}^*,$$

$$\rho_{N-1;(N-1)N} = f_{N-1}f_{(N-1)N}^* = \sum_{\substack{k,n,m=1 \\ m>n}}^{N_S} p_{N-1;k}p_{(N-1)N;n}^*a_k a_{nm}^*, \quad (14)$$

$$\rho_{N;N} = f_N f_N^* + \sum_{i=1}^{N-2} f_{iN} f_{iN}^* = \sum_{n,m=1}^{N_S} p_{N;n}p_{N;m}^*a_n a_m^* + \sum_{\substack{k,l,n,m=1 \\ m>n,l>k}}^{N_S} P_{NN;klm}a_{kl}a_{nm}^*, \quad (15)$$

$$\rho_{N;(N-1)N} = f_N f_{(N-1)N}^* = \sum_{\substack{k,n,m=1 \\ m>n}}^{N_S} p_{N;k}p_{(N-1)N;n}^*a_k a_{nm}^*, \quad (16)$$

$$\rho_{(N-1)N;(N-1)N} = f_{(N-1)N} f_{(N-1)N}^* = \sum_{\substack{k,l,n,m=1 \\ l>k,m>n}}^{N_S} P_{(N-1)N;kl}p_{(N-1)N;n}^*a_{kl}a_{nm}^*. \quad (17)$$

Here we introduce the following notations:

$$P_{N-1;knm} = \sum_{i=1}^{N-2} p_{i;k}p_{i(N-1);nm}^*, \quad P_{N;knm} = \sum_{i=1}^{N-2} p_{i;k}p_{iN;n}^*, \quad (18)$$

$$P_{(N-1)(N-1);klm} = \sum_{i=1}^{N-2} p_{i(N-1);kl}p_{i(N-1);nm}^*,$$

$$P_{(N-1)N;klm} = \sum_{i=1}^{N-2} p_{i(N-1);kl}p_{iN;n}^*, \quad P_{NN;klm} = \sum_{i=1}^{N-2} p_{iN;kl}p_{iN;n}^*.$$

Formulas (9-17) show that the receiver's density matrix depends on the set of parameters

$$\mathcal{P} = \{p_{N;i}, \quad p_{N-1;i}, \quad p_{(N-1)N;n}, \quad P_{N-1;inm}, \quad P_{N;inm}, \quad (19)$$

$$P_{(N-1)(N-1);klm}, \quad P_{(N-1)N;klm}, \quad P_{NN;klm},$$

$$i = 1, \dots, N_S, \quad k, n = 1, \dots, N_S - 1, \quad l, m = 2, \dots, N_S, \quad k > l, \quad m > n\},$$

which represent $\frac{1}{4}(3N_S^2 - 5N_S + 6)(N_S + 1)N_S$ complex characteristics of the transmission line depending on the particular Hamiltonian (governing the spin dynamics), the length of

the communication line and the time instant t . There is the following symmetry among these parameters:

$$P_{NN;klm} = P_{NN;nml}^*, \quad P_{(N-1)(N-1);klm} = P_{(N-1)(N-1;nml)}^*. \quad (20)$$

All parameters in \mathcal{P} can be directly defined for a given chain at any preferred time instant $t = t_0$ (see Sec.II C) and do not change during operation of the communication line.

It is important that the number of these parameters depends only on the dimensionality of the sender and receiver and does not depend on the length of the communication line. Below, for our convenience we use the notation P without subscript for a general element of the set \mathcal{P} : $P \in \mathcal{P}$.

C. Defining the characteristics (19) of the communication line

Although there are exact formulas (7) and (18) involving the Hamiltonian H and the time t , the actual values of these parameters for a given communication line can differ from the analytically calculated ones because of imperfections in the Hamiltonian. Therefore, before proceeding to operating a particular communication line, we have to define its parameters \mathcal{P} . For this purpose we create a set of preliminary states of the receiver using the following set of specially selected pure initial states of the sender:

$$a_0|0\rangle + a_i|i\rangle, \quad a_0^2 + a_k^2 = 1, \quad (21)$$

$$a_i|i\rangle + a_{nm}|nm\rangle, \quad a_k^2 + a_{nm}^2 = 1, \quad m > n \quad (22)$$

$$a_{kl}|kl\rangle + a_{nm}|nm\rangle, \quad a_{kl}^2 + a_{nm}^2 = 1, \quad m > n, \quad l > k, \quad (23)$$

$$a_{kl}|kl\rangle + ia_{nm}|nm\rangle, \quad a_{kl}^2 + a_{nm}^2 = 1, \quad m > n, \quad l > k, \quad (24)$$

$$i = 1, \dots, N_S, \quad l, m = 2, \dots, N_S, \quad k, n = 1, \dots, N_S - 1,$$

with some known values of the real constants a_k and a_{nm} . Then, the resulting states of the receiver will provide us with information about the set of parameters \mathcal{P} . This process can be considered the solution of a direct problem. On the contrary, finding the set of control parameters needed for the creation of a desirable receiver's state is an inverse problem and therefore is more tricky.

Now we proceed to defining the set of parameters \mathcal{P} using the formulas (9-17) for the elements of the density matrix (4) and the above set of initial conditions (21-24) at the

prescribed time instant $t = t_0$ (the way of fixing this time instant will be discussed in Sec.III).

Initial condition (21) yields (we give only the nonzero entries of the density matrix ρ^R in formulas (25-27))

$$\begin{aligned}\rho_{0;N-1} &= a_0 p_{N-1;k}^* a_k, \quad \rho_{0;N} = a_0 p_{N;k}^* a_k, \quad \rho_{N-1;N-1} = |p_{N-1;k}|^2 a_k^2, \\ \rho_{N-1;N} &= p_{N-1;k} p_{N;k}^* a_k^2, \quad \rho_{N;N} = |p_{N;k}|^2 a_k^2,\end{aligned}\tag{25}$$

defining $p_{N;k}$ and $p_{N-1;k}$.

Initial condition (22) yields

$$\begin{aligned}\rho_{0;N-1} &= P_{N-1;knm} a_k a_{nm}, \quad \rho_{0;N} = P_{N;knm} a_k a_{nm}, \\ \rho_{N-1;N-1} &= |p_{N-1;k}|^2 a_k^2 + P_{(N-1)(N-1);nmnm} a_{nm}^2, \quad \rho_{N-1;N} = p_{N-1;k} p_{N;k}^* a_k^2 + P_{(N-1)N;nmnm} a_{nm}^2, \\ \rho_{N-1;(N-1)N} &= p_{N-1;k} p_{(N-1)N;n timer}^* a_k a_{nm}, \quad \rho_{N;N} = |p_{N;k}|^2 a_k^2 + P_{NN;nmnm} a_{nm}^2, \\ \rho_{N;(N-1)N} &= p_{N;k} p_{(N-1)N;n timer} a_k a_{nm}, \quad \rho_{(N-1)N;(N-1)N} = |p_{(N-1)N;n timer}|^2 a_{nm}^2,\end{aligned}\tag{26}$$

defining $p_{(N-1)N;n timer}$, $P_{(N-1)N;nmnm}$, $P_{(N-1)(N-1);nmnm}$, $P_{NN;nmnm}$, $P_{N-1;knm}$ and $P_{N;knm}$.

Finally, initial condition (23) yields, in virtue of symmetry (20),

$$\begin{aligned}\rho_{N-1;N-1} &= P_{(N-1)(N-1);nmnm} a_{nm}^2 + P_{(N-1)(N-1);klkl} a_{kl}^2 + \\ &\quad 2\text{Re}(P_{(N-1)(N-1);klnm}) a_{kl} a_{nm}, \\ \rho_{N-1;N} &= P_{(N-1)N;nmnm} a_{nm}^2 + P_{(N-1)N;klkl} |a_{kl}|^2 + \\ &\quad 2\text{Re}(P_{(N-1)N;klnm}) a_{kl} a_{nm}, \\ \rho_{N;N} &= P_{NN;nmnm} a_{nm}^2 + P_{NN;klkl} a_{kl}^2 + 2\text{Re}(P_{NN;klnm}) a_{kl} a_{nm}, \\ \rho_{(N-1)N;(N-1)N} &= |p_{(N-1)N;n timer}|^2 a_{nm}^2 + |p_{(N-1)N;kl}|^2 a_{kl}^2 + \\ &\quad 2\text{Re}(p_{(N-1)N;kl} p_{(N-1)N;n timer}^*) a_{kl} a_{nm},\end{aligned}\tag{27}$$

defining the real parts of $P_{(N-1)(N-1);klnm}$, $P_{(N-1)N;klnm}$ and $P_{NN;klnm}$, $(kl) \neq (nm)$. The imaginary parts of these parameters can be obtained using the initial condition (24).

As a result, we obtain the whole list of parameters \mathcal{P} (19) for a given communication line at the prescribed time instant t_0 .

D. Creation of desired state. Approximate states.

In order to create the desired state A of the receiver, we have to solve the following system of algebraic quadratic equations (the inverse problem):

$$\rho_{ij}^R(\mathcal{P}, a) = A_{ij}, \quad i, j = 1, \dots, 4, j \geq i, \quad (28)$$

for the control parameters a_i, a_{ij} subject to the normalization condition (3). Here the elements of the receiver's density matrix ρ^R are given by formulas (9-17) and a is a list of all control parameters (8). If a solution of system (28,3) exists, then the state A may be exactly created using our communication line. However, in many cases we cannot create the state A exactly even if the system (28,3) is solvable. This may happen due to any of the following reasons.

1. The set of parameters \mathcal{P} cannot be exactly defined in general, see Sec.III B. So, system (28) must be replaced with the following one[26]:

$$\rho_{ij}^R(\mathcal{P}^{apr}, a) = A_{ij}, \quad i, j = 1, \dots, 4, j \geq i, \quad (29)$$

with approximate parameters \mathcal{P}^{apr} .

2. The set of control parameters a found as a solution to system (29,3) cannot be exactly implemented in the experiment, see Sec.III C. We implement the set a^{apr} which approximates the set of ideal parameters a up to some accuracy
3. The Hamiltonian contains imperfections (for instance, the coupling constants between nodes differ from the anticipated values), see Sec.IV.

Thus we have to estimate the difference between the matrix $\rho^R(\mathcal{P}, a^{apr})$ (in this formula, \mathcal{P} is the set of exact parameters because ρ^R is the matrix created in the experiment) and the desired matrix A . For this purpose we introduce the matrix discrepancy $\delta(\rho^R(\mathcal{P}, a^{apr}))$ by the following formula:

$$\delta(\rho^R(\mathcal{P}, a^{apr})) = \frac{\|\rho^R(\mathcal{P}, a^{apr}) - A\|}{\|A\|}, \quad (30)$$

where we use the standard matrix norm defined for a matrix x as follows

$$\|x\| = \sqrt{\sum_{i,j=1}^N |x_{ij}|^2} \quad (31)$$

If $\delta(\rho^R(\mathcal{P}, a^{apr})) < \varepsilon \ll 1$, then we say that the desired state A is created with the accuracy ε .

III. A PARTICULAR SPIN MODEL

A. Interaction Hamiltonian

The parameters (19) described in the previous section depend on the Hamiltonian governing the dynamics of a spin chain. Although the best result (the largest creatable space) is anticipated for the chain engineered for the perfect state transfer, we base our study on a simpler boundary-controlled model described by the nearest-neighbor XY Hamiltonian with two end-pairs of properly adjusted coupling constants:

$$\begin{aligned}
H = & \sum_{i=3}^{N-3} D(I_{ix}I_{(i+1)x} + I_{iy}I_{(i+1)y}) + \\
& \delta_1(I_{1x}I_{2x} + I_{1y}I_{2y} + I_{(N-1)x}I_{Nx} + I_{(N-1)y}I_{Ny}) + \\
& \delta_2(I_{2x}I_{3x} + I_{2y}I_{3y} + I_{(N-2)x}I_{(N-1)x} + I_{(N-2)y}I_{(N-1)y}),
\end{aligned} \tag{32}$$

where D , δ_1 and δ_2 are the coupling constants between the nearest neighbors, $I_{j\alpha}$ ($j = 1, \dots, N$, $\alpha = x, y, z$) is the j th spin projection on the α -axis. Below we put $D = 1$ which corresponds to using dimensionless time. The values of the coupling constants δ_i , $i = 1, 2$, are chosen in such a way that they maximize the probability $|p_{N;1}(t_0)|$ at some time instant t_0 , i.e. they maximize the value of the first maximum of $|p_{N;1}(t_0)|$. For instance, in the case of $N = 20, 60$ we have the following values of the parameters δ_i , of the maximized amplitude $|p_{N;1}(t_0)|$ and of the corresponding time instant t_0 :

$$\begin{aligned}
N = 20 : \quad & \delta_1 = 0.550, \quad \delta_2 = 0.817, \quad |p_{20;1}| = 0.99606, \quad t_0 = 26.441, \\
N = 60 : \quad & \delta_1 = 0.414, \quad \delta_2 = 0.720, \quad |p_{60;1}| = 0.99223, \quad t_0 = 70.203.
\end{aligned} \tag{33}$$

Formulas (33) complete the definition of the Hamiltonian (32) and allow us to uniquely define the set of parameters \mathcal{P} (19). We chose the dimensionality of the sender by the requirement that the number of real control parameters is not less than the number of creatable real parameters which is 15 for the two-qubit receiver. Thus, the minimal dimensionality of the sender is $N_S = 4$ (20 independent real control parameters in initial state (2)) which is used hereafter.

B. Classification of parameters (19) by absolute value

Parameters (19) differ by their absolute values which are determined by two main factors. First, the mirror symmetry in $p_{i;j}$, $p_{kl;nm}$ appearing in the definitions (18) of parameters \mathcal{P} . Due to the spatial symmetry of the chain the transition amplitudes (7) tend to be large when the sites or pairs of sites involved are at positions symmetric with respect to the center of the chain. Second, the position of nodes i and j in $p_{i;j}$ (or k, l, n and m in $p_{kl;nm}$): these quantities decrease with approaching the center of communication line. Thus, we separate the parameters into three families.

Family I consists of 13 parameters. For chains of $N = 20$ and 60 nodes the absolute values of these parameters are in the intervals $(0.9356, 0.9961)$ and $(0.8341, 0.9923)$, respectively, see Table I. The large values of these parameters are explained by the mirror symmetry of the transition amplitudes $p_{i;i}$ and $p_{nm;nm}$ which appear in definitions (18) of the parameters \mathcal{P} . For instance, the parameters $P_{19;3,2,3}$ and $P_{19,20;2,3,1,3}$ contain the large terms $p_{18;3}p_{19,18;2,3}$ and $p_{19,18;2,3}p_{20,18;1,3}$, respectively, and so on.

Family II consists of 14 parameters. For the chains of $N = 20$ and 60 nodes the absolute values of these parameters are in the intervals $(0.0635, 0.0893)$ and $(0.1616, 0.2165)$, respectively, see Table II. The mirror symmetry just mentioned is significantly reduced in these parameters (in definitions (18) of these parameters, there is no term consisting of a product of two symmetrical transition amplitudes).

The values of parameters \mathcal{P} are given with the accuracy $\sim 10^{-5}$ in both tables. The symmetry (20) is confirmed in Tables I and II. Equalities between some other elements in this table are approximate and disappear in higher order approximations.

Family III consists of 143 parameters with destroyed mirror symmetry and with absolute values below 0.0193 (for the chain $N = 20$) and below 0.0468 ($N = 60$). This family for the chain of 20 nodes is given in Appendix, Sec.VI.

By adjusting all of the coupling constants in the Hamiltonian (32), we may approach the chain engineered for perfect state transfer. In this limiting case the transition amplitudes (7) are equal to unity if $k = N + 1 - i$ ($(n, m) = (N + 1 - i, N + 1 - j)$) [4] and thus all parameters from Family I tend to unity, while all other parameters vanish.

$P \in \mathcal{P}$	\mathcal{N}	$N = 20$	$N = 60$
$p_{N-1;2}$	1	0.96743 <i>i</i>	0.91422 <i>i</i>
$p_{N;1}$	2	0.99606 <i>i</i>	0.99223 <i>i</i>
$p_{(N-1)N;1,2}$	3	0.96361	0.90711
$P_{N-1;3,2,3}$	4	0.96707 <i>i</i>	0.91238 <i>i</i>
$P_{N-1;4,2,4}$	5	0.96741 <i>i</i>	0.91422 <i>i</i>
$P_{N;3,1,3}$	6	0.99601 <i>i</i>	0.99220 <i>i</i>
$P_{N;4,1,4}$	7	0.98812 <i>i</i>	0.94575 <i>i</i>
$P_{NN;1,3,1,3}$	8	0.99246	0.98649
$P_{NN;1,4,1,4}$	9	0.98424	0.93841
$P_{(N-1)(N-1);2,3,2,3}$	10	0.93561	0.83415
$P_{(N-1)(N-1);2,4,2,4}$	11	0.94387	0.88263
$P_{(N-1)N;2,3,1,3}$	12	0.96361	0.90711
$P_{(N-1)N;2,4,1,4}$	13	0.96361	0.90711

TABLE I: Family I of parameters \mathcal{P} (19); $0.9356 < |P| < 0.9961$ and $0.8341 < |P| < 0.9923$ for chains of $N = 20$ and $N = 60$ nodes, respectively.

$P \in \mathcal{P}$	\mathcal{N}	$N = 20$	$N = 60$
$p_{N-1;4}$	1	-0.08929 <i>i</i>	-0.21641 <i>i</i>
$p_{(N-1)N;1,4}$	2	-0.08894	-0.21473
$P_{N-1;2,2,4}$	3	0.08929 <i>i</i>	0.21641 <i>i</i>
$P_{N-1;3,3,4}$	4	0.08925 <i>i</i>	0.21598 <i>i</i>
$P_{N;2,1,2}$	5	0.06384 <i>i</i>	0.16292 <i>i</i>
$P_{N;4,1,2}$	6	0.08604 <i>i</i>	0.19631 <i>i</i>
$P_{N;2,1,4}$	7	0.08604 <i>i</i>	0.19631 <i>i</i>
$P_{NN;1,2,1,2}$	8	0.06358	0.16166
$P_{NN;1,4,1,2}$	9	0.08570	0.19479
$P_{NN;1,2,1,4}$	10	0.08570	0.19479
$P_{(N-1)(N-1);3,4,2,3}$	11	0.08635	0.19745
$P_{(N-1)(N-1);2,3,3,4}$	12	0.08635	0.19745
$P_{(N-1)N;2,4,1,2}$	13	0.08894	0.21473
$P_{(N-1)N;3,4,1,3}$	14	0.08894	0.21473

TABLE II: Family II of parameters \mathcal{P} (19); $0.0635 < |P| < 0.0893$ and $0.1616 < |P| < 0.2165$ for chains of $N = 20$ and $N = 60$ nodes, respectively.

C. Example of remote state creation: Werner state

As a particular example we use a chain of 20 nodes to create the Werner state [25] which in the two-qubit case reads as follows:

$$\rho^W = \begin{pmatrix} \frac{1-p}{4} & 0 & 0 & 0 \\ 0 & \frac{1+p}{4} & -\frac{p}{2} & 0 \\ 0 & -\frac{p}{2} & \frac{1+p}{4} & 0 \\ 0 & 0 & 0 & \frac{1-p}{4} \end{pmatrix}, \quad 0 \leq p \leq 1. \quad (34)$$

To create the zero entries in this matrix with our tool of four node sender we have to put $a_i = 0$, $i = 0, 1, \dots, 4$. The system of equations (29) for the control parameters a_{ij} can be written equating the corresponding elements of the density matrices $\rho^R(\mathcal{P}^{apr}, a)$ and ρ^W

(we use the parameters \mathcal{P}^{apr} from the Tables I and II and from the Appendix):

$$\sum_{\substack{k,l,n,m=1 \\ l>k, m>n}}^{N_S} P_{(N-1)N;kl} P_{(N-1)N;nm}^* a_{kl} a_{nm}^* = \frac{1-p}{4}, \quad (35)$$

$$\sum_{\substack{k,l,n,m=1 \\ m>n, l>k}}^{N_S} P_{(N-1)(N-1);klm} a_{kl} a_{nm}^* = \sum_{\substack{k,l,n,m=1 \\ m>n, l>k}}^{N_S} P_{NN;klm} a_{kl} a_{nm}^* = \frac{1+p}{4}, \quad (36)$$

$$\sum_{\substack{k,l,n,m=1 \\ m>n, l>k}}^{N_S} P_{(N-1)N;klm} a_{kl} a_{nm}^* = -\frac{p}{2}. \quad (37)$$

In turn, eq. (37) splits into the real and imaginary parts:

$$\text{Im} \left(\sum_{\substack{k,l,n,m=1 \\ m>n, l>k}}^{N_S} P_{(N-1)N;klm} a_{kl} a_{nm}^* \right) = 0, \quad \text{Re} \left(\sum_{\substack{k,l,n,m=1 \\ m>n, l>k}}^{N_S} P_{(N-1)N;klm} a_{kl} a_{nm}^* \right) = -\frac{p}{2}. \quad (38)$$

In addition, we have the normalization condition (3) for the control parameters which now reads:

$$\sum_{\substack{i,j=1 \\ j>i}}^4 |a_{ij}|^2 = 1. \quad (39)$$

All in all, we have a system of 6 real equations (35,36,38,39), so that we need 6 real control parameters to solve it. Consequently, since we are interested in particular solutions, we may consider only the real parameters a_{ij} . The numerical investigation of this system shows that it is solvable for a_{ij} if p is in the interval $0 \leq p \leq 0.8744$. We collect the results of the calculations for the parameter values $p = 0.1n$, $n = 0, 1, \dots, 8$, in Table III to five digit accuracy (the same accuracy is used for the parameters \mathcal{P} in Sec.III B). The appropriate discrepancies $\delta(\rho^R)$ are given in the last column.

There is one more simplification of the model which follows from the fact that the absolute values of the parameters \mathcal{P} from Family III are small and thus we can put all of them equal to zero as the first approximation. Solving the system (35,36,38,39) for a_{ij} in this case we obtain new values for the control parameters collected in in Table IV together with appropriate discrepancies $\delta(\rho^R)$ (the last column).

IV. EFFECT OF IMPERFECTIONS IN THE CHAIN

Considering a set of chains of equal length governed by the same Hamiltonian H we can expect minor differences between them owing to the unavoidable imperfections in the

p	a_{12}	a_{13}	a_{14}	a_{23}	a_{24}	a_{34}	$\delta(\rho^R)$
0	0.53233	0.42337	0.20146	0.21791	-0.44593	0.50046	4.235×10^{-5}
0.1	0.52208	0.24658	0.41283	0.38198	-0.34211	0.48296	2.641×10^{-5}
0.2	0.49965	0.14204	0.48397	0.42820	-0.32433	0.45540	1.532×10^{-5}
0.3	0.47309	0.07861	0.52371	0.44462	-0.34473	0.42334	3.883×10^{-5}
0.4	0.44366	0.03642	0.55344	0.44645	-0.38257	0.38713	1.744×10^{-6}
0.5	0.41131	0.01047	0.57913	0.43460	-0.43236	0.34571	2.066×10^{-5}
0.6	0.37550	0.00361	0.60326	0.40331	-0.49433	0.29673	2.347×10^{-5}
0.7	0.33507	0.02962	0.62617	0.33516	-0.57203	0.23496	6.332×10^{-6}
0.8	0.28714	0.13374	0.63657	0.17462	-0.66556	0.14487	1.604×10^{-5}

TABLE III: A particular solution of system (35,36,38,39) with the accuracy 10^{-5} for different p , all parameters \mathcal{P} are taken into account . The last column shows the values of the discrepancy $\delta(\rho^R(a))$ between the created state and the Werner state.

p	a_{12}	a_{13}	a_{14}	a_{23}	a_{24}	a_{34}	$\delta(\rho^R)$
0	0.55924	0.00127	0.43728	0.47083	-0.00135	0.52378	2.906×10^{-2}
0.1	0.53510	-0.04674	0.46417	0.49567	-0.05174	0.49766	2.893×10^{-2}
0.2	0.50894	-0.09432	0.48584	0.51641	-0.09569	0.46926	2.760×10^{-2}
0.3	0.48066	-0.13718	0.50409	0.53327	-0.14033	0.43815	2.540×10^{-2}
0.4	0.44449	-0.30614	0.46113	0.57393	-0.04057	0.40624	2.234×10^{-2}
0.5	0.41591	-0.22160	0.53091	0.55857	-0.22518	0.36516	1.964×10^{-2}
0.6	0.37799	-0.26409	0.53974	0.56726	-0.26749	0.32050	1.648×10^{-2}
0.7	0.33452	-0.30791	0.54518	0.57314	-0.31105	0.26614	1.317×10^{-2}
0.8	0.28247	-0.35464	0.54622	0.57594	-0.35749	0.19123	9.600×10^{-3}

TABLE IV: A particular solution of system (35,36,38,39) with the accuracy 10^{-5} for different p , Parameters \mathcal{P} from Family III have been set equal to zero . The last column shows the discrepancy $\delta(\rho^R)$ between the created state and the Werner state. Notice that setting the parameters from Family III equal to zero leads to significant changes in the results, as compared to Table III. This happens because we deal with particular solutions in both cases.

preparation of these chains. Consequently, the parameters \mathcal{P} (19) vary slightly passing from one chain to another. Although the parameters \mathcal{P} can be defined for any of these chains using the protocol of Sec.II C, the procedure is rather complicated. Instead we can calculate the parameters \mathcal{P} for one of these chains and take into account the possible variations of these parameters in working with other chains. In this way, we, generally speaking, prepare an approximate state instead of the exactly required one. However, this can be done if the approximate state is acceptable for our purposes.

Thus, below we assume that only the two pairs of the coupling constants δ_1 and δ_2 close to the ends of the chain in Hamiltonian (32) are perfectly adjusted. All other coupling constants are only manufactured with a certain accuracy ε , i.e.

$$D_i = 1 + \varepsilon \Delta_i, \quad i = 3, 4, \dots, N-3, \quad (40)$$

where ε is the amplitude of random perturbations and Δ_i are random numbers in the interval $-1 \leq \Delta_i \leq 1$. Below we present results for the randomness amplitudes $\varepsilon = 0.025$ and $\varepsilon = 0.05$.

A. Deviations of parameters (19) due to imperfections of the Hamiltonian

To characterize the effect of random imperfections of the Hamiltonian, we consider N_p independent chains of the same type and characterize the parameters \mathcal{P} associated with these chains by their mean values and standard deviations. Thus, for some parameter $P \in \mathcal{P}$, the mean value $\langle P \rangle$ and standard deviation $\sigma(P)$ are calculated by the usual formulas:

$$\langle P(\varepsilon) \rangle = \frac{1}{N_p} \sum_{i=1}^{N_p} P_i(\varepsilon), \quad (41)$$

$$\sigma(P(\varepsilon)) = \sqrt{\frac{1}{N_p - 1} \sum_{i=1}^{N_p} (P_i(\varepsilon) - \langle P(\varepsilon) \rangle)^2}, \quad (42)$$

where P_i is the value of P for the i th chain.

As a result, the value $P(\varepsilon)$ can be approximated as

$$P(\varepsilon) = \langle P(\varepsilon) \rangle \pm \sigma(P(\varepsilon)). \quad (43)$$

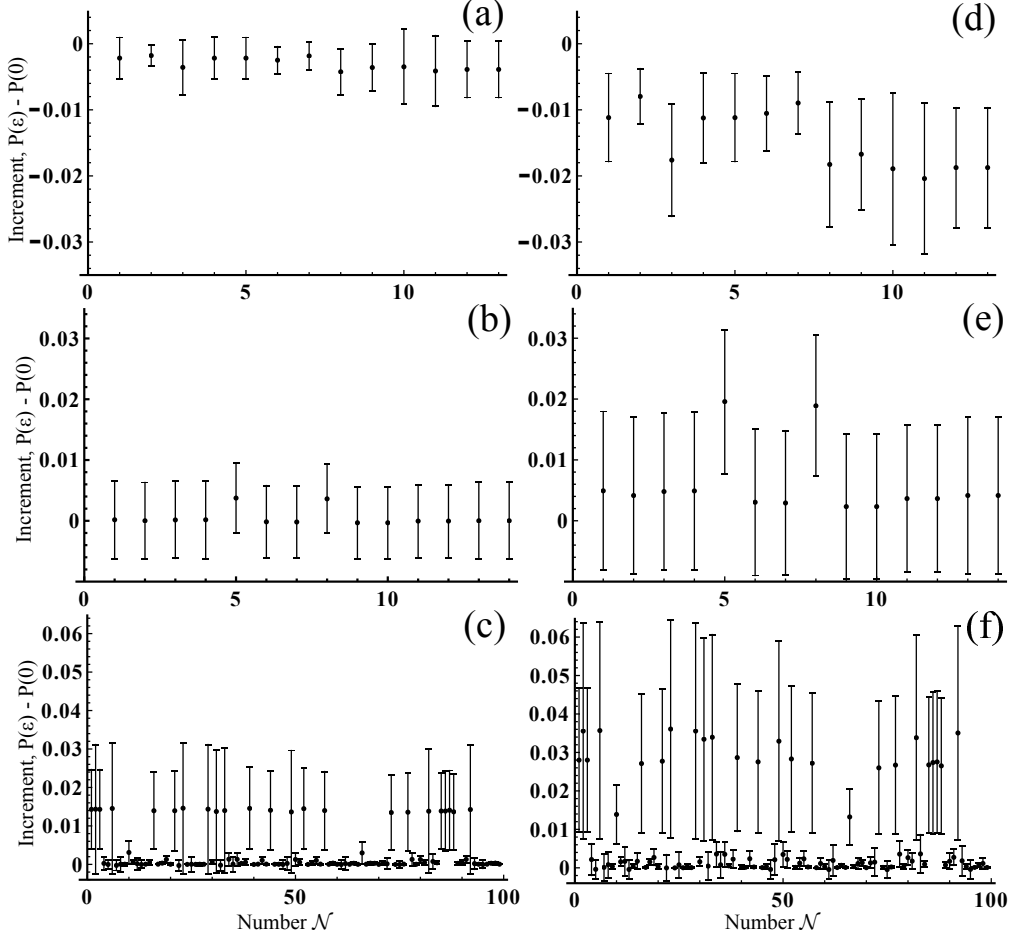


FIG. 1: Quantity $P(\varepsilon) - P(0)$ with $P(\varepsilon)$ given in (43) for three families of the parameters \mathcal{P} and two values of ε . Averaging is over $N_p = 100$ random chains. Numbers \mathcal{N} enumerate the parameters \mathcal{P} in accordance with Tables I, II and Appendix. Notice that the scales are different in these figures. The mean values $\langle P(\varepsilon) \rangle$ are shown by the bold dots, the vertical bars indicate the standard deviations $\sigma(P(\varepsilon))$. (a,b,c) $\varepsilon = 0.025$; (d,e,f) $\varepsilon = 0.05$. (a,d) Family I, (b,e) Family II, (c,f) Family III of parameters \mathcal{P} .

The quantities $P(\varepsilon) - P(0)$ ($P(0)$ denotes parameter obtained in Sec.III B for the unperturbed Hamiltonian) are represented in Fig.1 for the three families I, II, and III of parameters \mathcal{P} and for the two values of the randomness amplitude $\varepsilon = 0.025, 0.05$, averaging over $N_p = 100$ chains. Fig.1 shows that the mean values $\langle P(\varepsilon) \rangle$ of the parameters from the Family I are less than the ideal values $P(0)$ of these parameters, while the mean values of the parameters from the Family II almost coincide with their ideal values. The parameters from the Family III are separated into two parts: the mean values of some of them almost

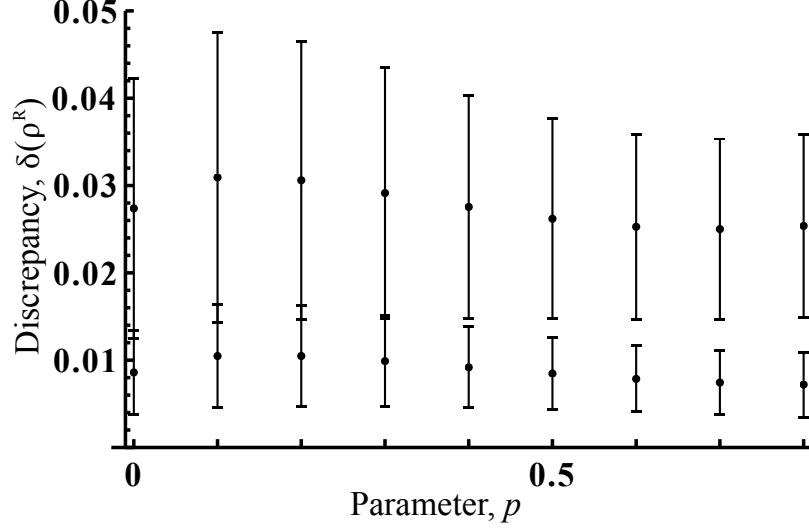


FIG. 2: Density matrix discrepancy $\delta(\rho^R(\varepsilon)) = \langle \delta(\rho^R(\varepsilon)) \rangle \pm \sigma(\delta(\rho^R(\varepsilon)))$ for different values of the parameter p of the Werner state. The averaging is over $N_p = 100$ random chains. The mean values $\langle \delta(\rho^R(\varepsilon)) \rangle$ are shown by the bold dots, the vertical bars indicate the standard deviations $\sigma(\delta(\rho^R(\varepsilon)))$. The upper and lower sets of dots correspond to $\varepsilon = 0.05$ and $\varepsilon = 0.025$, respectively.

coincide with their ideal values, while the mean values of others differ from the ideal values rather significantly. Fig.1 shows also that the deviation from the calculated mean values is almost uniform for the parameters from the Family II. We may conclude, that the imperfections of 5% ($\varepsilon = 0.05$) lead to reasonable approximations of the parameters \mathcal{P} with roughly the same accuracy.

B. Creation of the Werner state using a chain of 20 nodes with imperfections

The parameters a calculated in Sec.III C were determined for the chain with unperturbed coupling constants. In this section we use these parameters to create the Werner state using the chain of 20 nodes with imperfections and characterize the effect of imperfections by the mean value of the density matrix discrepancy, $\langle \delta(\rho^R(\varepsilon)) \rangle$, and its standard deviation $\sigma(\delta(\rho^R(\varepsilon)))$. Results of our calculations are shown in Fig.2. From this figure we see that the imperfection $\varepsilon = 0.025$ yields discrepancy $\delta \lesssim 0.02$ and imperfection $\varepsilon = 0.05$ yields discrepancy $\delta \lesssim 0.05$ (compare with the discrepancy δ in the unperturbed case, Table III), i.e. the considered values of imperfection strength yield a reasonable approximation of the density matrix. This figure shows also that both the mean value $\langle \delta \rangle$ and the standard deviation $\sigma(\delta)$ depend only slightly on the parameter p of the Werner state, i.e., the effect of

imperfections of the Hamiltonian on the realization of Werner states is almost independent of p .

V. CONCLUSIONS

We consider the problem of remote creation of a two-qubit state using a two-qubit excitation initial state of the sender. The number of qubits of the sender is not important for the protocol, although it affects the creatable region of the receiver's state space. We show that a given communication line with sender and receiver of fixed size has a certain number of parameters \mathcal{P} (19) which completely characterize the Hamiltonian dependence and evolution of the receiver's state. They are attributes of the communication line and do not change during the operation. These parameters are responsible for the volume of the creatable region of the receiver's state-space. Another set of parameters characterizes the sender's initial state. This set is Hamiltonian independent and is referred to as a set of control parameters (8). These are the parameters responsible for creation of a particular state from the creatable region.

When dealing with a particular communication line, we can define the parameters \mathcal{P} using the analytical formulas of Sec.II B. However, these formulas may give improper results if the actual parameters of the chain differ from their theoretically prescribed values. Thus we recommend to define these parameters performing a set of remote state-creation processes with prescribed initial conditions (the direct problem), Sec.II C. These parameters can be defined only with some accuracy, which is a reason of dealing with the approximate remote state creation in reality. Another reason is that the analytically calculated control parameters can not be implemented with absolute accuracy. Thus, dealing with approximately created states we have to estimate the accuracy of state creation. For this purpose the density matrix discrepancy $\delta(\rho^R)$ is introduced in Sec.II D.

Not all parameters \mathcal{P} are equally important. In our examples of $N = 20$ and 60 nodes (Sec.III B) we separate them into three families by their absolute values. The most significant is the first family of 13 parameters. These are the parameters which remain (and are equal to unity) in the chain engineered for perfect state transfer, while all other parameters vanish in this chain.

Since determining the parameters \mathcal{P} for any particular chain is a rather complicated

procedure, we may not need to calculate them separately for all chains of the same type (i.e. governed by the same Hamiltonian and having the same length). Instead, we calculate them for one of these chains, then the effect of random imperfections of the Hamiltonian can be reflected in the accuracy of the state creation, if only this accuracy is acceptable. We estimate the effect of imperfections on some quantity x by its mean value $\langle x \rangle$ and standard deviation $\sigma(x)$.

All these theoretical arguments have been justified by the example of remote creation of the two-qubit Werner state with parameter p in the interval $0 \leq p \leq 0.8744$ in Secs.III C and IV B using the chain of 20 nodes and two amplitudes of the Hamiltonian imperfection (deviation of coupling constants) $\varepsilon = 0.025, 0.05$.

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VI. APPENDIX. FAMILY III OF PARAMETERS (19) IN MODEL OF SEC. III
WITH ABSOLUTE VALUES < 0.02 , $N = 20$

$P \in \mathcal{P}$	\mathcal{N}	$N = 20$
$p_{19;1}$	1	-1.007×10^{-4}
$p_{19;3}$	2	-7.168×10^{-3}
$p_{20;2}$	3	-1.007×10^{-4}
$p_{20;3}$	4	$-1.928 \times 10^{-2}i$
$p_{20;4}$	5	-3.860×10^{-3}
$p_{19,20;1,3}$	6	$7.142 \times 10^{-3}i$
$p_{19,20;2,3}$	7	1.865×10^{-2}
$p_{19,20;2,4}$	8	$-3.743 \times 10^{-3}i$
$p_{19,20;3,4}$	9	1.749×10^{-3}
$P_{19;1,1,2}$	10	$-7.606 \times 10^{-3}i$
$P_{19;2,1,2}$	11	3.665×10^{-6}
$P_{19;3,1,2}$	12	$-1.858 \times 10^{-2}i$
$P_{19;4,1,2}$	13	-3.720×10^{-3}
$P_{19;1,1,3}$	14	-5.442×10^{-5}
$P_{19;2,1,3}$	15	$7.194 \times 10^{-7}i$
$P_{19;3,1,3}$	16	-3.695×10^{-5}
$P_{19;4,1,3}$	17	$2.757 \times 10^{-5}i$
$P_{19;1,1,4}$	18	$7.023 \times 10^{-4}i$
$P_{19;2,1,4}$	19	8.958×10^{-6}
$P_{19;3,1,4}$	20	$1.714 \times 10^{-3}i$
$P_{19;4,1,4}$	21	4.440×10^{-4}
$P_{19;1,2,3}$	22	$1.858 \times 10^{-2}i$
$P_{19;2,2,3}$	23	-7.170×10^{-3}
$P_{19;4,2,3}$	24	-7.199×10^{-5}

$P \in \mathcal{P}$	\mathcal{N}	$N = 20$
$P_{19;1,2,4}$	25	-3.729×10^{-3}
$P_{19;3,2,4}$	26	7.216×10^{-5}
$P_{19;1,3,4}$	27	$1.742 \times 10^{-3}i$
$P_{19;2,3,4}$	28	-1.762×10^{-7}
$P_{19;4,3,4}$	29	7.161×10^{-3}
$P_{20;1,1,2}$	30	-3.665×10^{-6}
$P_{20;3,1,2}$	31	6.907×10^{-3}
$P_{20;1,1,3}$	32	$1.928 \times 10^{-2}i$
$P_{20;2,1,3}$	33	-6.909×10^{-3}
$P_{20;4,1,3}$	34	6.377×10^{-4}
$P_{20;1,1,4}$	35	-3.869×10^{-3}
$P_{20;3,1,4}$	36	-6.375×10^{-4}
$P_{20;1,2,3}$	37	1.878×10^{-6}
$P_{20;2,2,3}$	38	$1.236 \times 10^{-3}i$
$P_{20;3,2,3}$	39	2.344×10^{-4}
$P_{20;4,2,3}$	40	$1.665 \times 10^{-3}i$
$P_{20;1,2,4}$	41	$3.771 \times 10^{-7}i$
$P_{20;2,2,4}$	42	-2.387×10^{-4}
$P_{20;3,2,4}$	43	$2.683 \times 10^{-5}i$
$P_{20;4,2,4}$	44	-2.335×10^{-4}
$P_{20;1,3,4}$	45	1.762×10^{-7}
$P_{20;2,3,4}$	46	$-1.692 \times 10^{-3}i$
$P_{20;3,3,4}$	47	-3.848×10^{-3}
$P_{20;4,3,4}$	48	$-1.912 \times 10^{-2}i$

$P \in \mathcal{P}$	\mathcal{N}	$N = 20$
$P_{20,20;1,2,1,3}$	49	$6.880 \times 10^{-3}i$
$P_{20,20;1,3,1,4}$	50	$7.096 \times 10^{-4}i$
$P_{20,20;1,2,2,3}$	51	1.231×10^{-3}
$P_{20,20;1,3,2,3}$	52	$-2.335 \times 10^{-4}i$
$P_{20,20;1,4,2,3}$	53	1.659×10^{-3}
$P_{20,20;2,3,2,3}$	54	2.385×10^{-5}
$P_{20,20;1,2,2,4}$	55	$2.377 \times 10^{-4}i$
$P_{20,20;1,3,2,4}$	56	2.673×10^{-5}
$P_{20,20;1,4,2,4}$	57	$2.326 \times 10^{-4}i$
$P_{20,20;2,3,2,4}$	58	$4.604 \times 10^{-6}i$
$P_{20,20;2,4,2,4}$	59	8.978×10^{-7}
$P_{20,20;1,2,3,4}$	60	-1.685×10^{-3}
$P_{20,20;1,3,3,4}$	61	$3.832 \times 10^{-3}i$
$P_{20,20;1,4,3,4}$	62	-1.905×10^{-2}
$P_{20,20;2,3,3,4}$	63	-3.300×10^{-5}
$P_{20,20;2,4,3,4}$	64	$4.605 \times 10^{-6}i$
$P_{20,20;3,4,3,4}$	65	3.835×10^{-4}
$P_{19,19;1,2,1,2}$	66	7.358×10^{-3}
$P_{19,19;1,2,1,3}$	67	$-5.265 \times 10^{-5}i$
$P_{19,19;1,3,1,3}$	68	3.864×10^{-7}
$P_{19,19;1,2,1,4}$	69	-6.795×10^{-4}
$P_{19,19;1,3,1,4}$	70	$-4.862 \times 10^{-6}i$
$P_{19,19;1,4,1,4}$	71	6.275×10^{-5}
$P_{19,19;1,2,2,3}$	72	-1.797×10^{-2}

$P \in \mathcal{P}$	\mathcal{N}	$N = 20$
$P_{19,19;1,3,2,3}$	73	$-3.574 \times 10^{-5}i$
$P_{19,19;1,4,2,3}$	74	1.659×10^{-3}
$P_{19,19;1,2,2,4}$	75	$-3.598 \times 10^{-3}i$
$P_{19,19;1,3,2,4}$	76	2.673×10^{-5}
$P_{19,19;1,4,2,4}$	77	$4.304 \times 10^{-4}i$
$P_{19,19;2,3,2,4}$	78	$-7.098 \times 10^{-4}i$
$P_{19,19;1,2,3,4}$	79	-1.685×10^{-3}
$P_{19,19;1,3,3,4}$	80	$-3.497 \times 10^{-6}i$
$P_{19,19;1,4,3,4}$	81	1.563×10^{-4}
$P_{19,19;2,4,3,4}$	82	$-6.928 \times 10^{-3}i$
$P_{19,19;3,4,3,4}$	83	8.021×10^{-3}
$P_{19,20;1,2,1,2}$	84	$2.884 \times 10^{-6}i$
$P_{19,20;1,3,1,3}$	85	$-3.785 \times 10^{-5}i$
$P_{19,20;1,4,1,4}$	86	$4.450 \times 10^{-4}i$

$P \in \mathcal{P}$	\mathcal{N}	$N = 20$
$P_{19,20;2,3,2,3}$	87	$-2.356 \times 10^{-4}i$
$P_{19,20;2,4,2,4}$	88	$2.472 \times 10^{-4}i$
$P_{19,20;3,4,3,4}$	89	$2.065 \times 10^{-4}i$
$P_{19,20;1,3,1,2} = P_{19,20;3,4,2,4}$	90	7.220×10^{-7}
$P_{19,20;1,4,1,2} = P_{19,20;3,4,2,3}^*$	91	$8.994 \times 10^{-6}i$
$P_{19,20;2,3,1,2} = P_{19,20;3,4,1,4}^*$	92	$-7.140 \times 10^{-3}i$
$P_{19,20;1,2,1,3} = P_{19,20;2,4,3,4}$	93	-1.865×10^{-2}
$P_{19,20;1,4,1,3} = P_{19,20;2,4,2,3}$	94	1.721×10^{-3}
$P_{19,20;1,2,1,4} = P_{19,20;2,3,3,4}^*$	95	$-3.734 \times 10^{-3}i$
$P_{19,20;1,3,1,4} = P_{19,20;2,3,2,4}$	96	2.767×10^{-5}
$P_{19,20;1,2,2,3} = P_{19,20;1,4,3,4}^*$	97	$1.942 \times 10^{-6}i$
$P_{19,20;1,3,2,3} = P_{19,20;1,4,2,4}$	98	1.015×10^{-8}
$P_{19,20;1,2,2,4} = P_{19,20;1,3,3,4}$	99	-3.888×10^{-7}

We also observe a symmetry among the parameters $P_{12,20;klmn}$ in lines $\mathcal{N} = 90, \dots, 99$ of the above Table. This is an “approximate” symmetry which is destroyed in higher order approximation. In addition, $P_{19,20;klmn} = 0$, $k \neq l \neq n \neq m$. Both of these facts appear due to the assumption of nearest-neighbor couplings in Hamiltonian (32) and the mirror symmetry of the chain.

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- [26] Obviously, the matrix $\rho_{ij}^R(\mathcal{P}^{apr}, a)$ in eq.(29) in general is not a density matrix. However, this is not important, because this system serves only to define the parameters a which will be im-

plemented in the initial state of the sender. Then the matrix $\rho_{ij}^R(\mathcal{P}, a)$ obtained after evolution of the implemented initial state is a correct density matrix by construction, and this matrix is used in formula (30) introducing the matrix discrepancy.